PRINCIPAL CHARACTERISTICS OF TURBULENT NONLINEAR STOKES-FLUID FLOWS WITH TRANSVERSE SHEAR

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The calculation of the principal mean-flow characteristics and the distribution of the fluctuating quantities are examined for the case of turbulent flows of nonlinear Stokes fluids with a given characteristic equation.

The most general form of the characteristic equation for fluidity may be written as [1–3]

$$\tau_{ij} = \alpha\delta_{ij} + \beta\dot{e}_{ij} + \gamma\dot{e}_{ik}\dot{e}_{kj}, \tag{1}$$

where α, β, and γ are functions of the three invariants of the strain-rate tensor and the thermodynamic state.

The deformation behavior of any medium must satisfy certain conditions. A medium whose characteristic equation satisfies the Stokes postulates is usually called a Stokes fluid. Stokes fluids do not exhibit "memory." For these fluids Eq. (1) has the form

$$\tau_{ij} = (-p + \alpha^*)\delta_{ij} + \beta\dot{e}_{ij} + \gamma\dot{e}_{ik}\dot{e}_{kj}, \tag{2}$$

where p is the thermodynamic pressure, and α^* is a certain function of the invariants of the strain-rate tensor, which in accordance with Stokes' fourth postulate [3] vanishes at $e_{ij} = 0$. In the case of incompressible viscous fluids p is indeterminate and, consequently, any definition of pressure is valid that does not contradict Stokes' fourth postulate. Usually, p is assumed to coincide [4] with the mean pressure $\bar{p} = -I_{1T}/3$, with p and $\bar{p}$ differing significantly only in such rapidly developing processes as, for example, explosions. For the case of incompressible fluids Eq. (2) gives the relation for the pressure and mean pressure

$$(p - 3\bar{p}) = \gamma\dot{e}_{ik}\dot{e}_{ki}, \tag{3}$$

whence it is clear that $p = \bar{p}$ if and only if $\gamma = 0$. Thus, by adopting the hypothesis that $p = \bar{p}$, we confine ourselves to the case of a quasi-linear relation between the stress tensor and the strain-rate tensor. The coefficient β for an incompressible fluid can be represented in the form of a power series in the second invariant

$$\beta = \beta_0 + \beta_1 I_2 + \beta_2 I_2^2 + \ldots . \tag{4}$$

Taking the first two terms of the expansion only (thereby limiting the region of shear values considered) and using customary notation,

$$\beta = \mu - \mu_2 I_2. \tag{5}$$

Thus, the characteristic equation, which we will continue to use and whose region of applicability can be determined only by experiment, becomes

$$\tau_{ij} = -p\,\delta_{ij} + (\mu - \mu_2 I_2)\dot{e}_{ij}, \tag{6}$$

where the dimensions of μ are M/LT and of μ_2 are MT/L.

Equation (6), which describes the flow of pseudoplastic and dilatant fluids, was used in [5] to investigate stability.

To close the system of Reynolds equations, as distinct from the phenomenological Prandtl-Boussinesq theory, we include the equations for the variation of the Reynolds stresses. This permits a detailed examination of the fluctuating-motion characteristics and its effect on the mean motion. We use the Reynolds stress equations, reduced to second moment balance equations by introducing certain approximations based on Kolmogorov's ideas [7]. This or a similar method was used successfully to calculate the characteristics of turbulent flows in pipes and channels,* boundary layers [17], two-phase flows [12], and magnetohydrodynamic flows [13–16]. Its great advantage is that it can readily be extended to the calculation of the characteristics of turbulent flows when the turbulence is influenced by various external factors.

The method of obtaining the Reynolds stress equations is well known [8]. Using Eq. (6) we obtain the following system of equations in a Cartesian coordinate system**

$$\rho\left[\frac{\partial\overline{u_iu_j}}{\partial t} + \bar{U}_a\frac{\partial\overline{u_iu_j}}{\partial x_a} + \overline{u_ju_a}\frac{\partial}{\partial x_a}\times\right.$$
$$\times\left(\bar{U}_i - \frac{1}{\rho}\mu_2\frac{\partial\bar{I}_2}{\partial x_i}\right) + \overline{u_iu_a}\frac{\partial}{\partial x_a}\left(\bar{U}_j - \frac{1}{\rho}\mu_2\frac{\partial\bar{I}_2}{\partial x_j}\right) -$$
$$-\overline{p_*\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} + \overline{\left(\frac{\partial u_jp_*}{\partial x_i} + \frac{\partial u_ip_*}{\partial x_j}\right)} -$$
$$-\frac{\partial}{\partial x_a}\left[(\mu - \mu_2\bar{I}_2)\frac{\partial\overline{u_iu_j}}{\partial x_a}\right] +$$
$$+\rho\frac{\partial}{\partial x_a}\overline{u_iu_ju_a} + 2(\mu - \mu_2\bar{I}_2)\overline{\frac{\partial u_i}{\partial x_a}\frac{\partial u_j}{\partial x_a}} -$$
$$-\overline{(u_jf_i + u_if_j)} = 0, \tag{7}$$

where

$$i,\ j,\ a = 1,\ 2,\ 3;\quad p_* = p + \mu_2\frac{\partial\bar{I}_2}{\partial x_a}u_a.$$

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*These studies are listed in the monograph by Monin and Yaglom [8], and in papers by Rotta [9] and Levin [10, 11].

**Following Lumley [6], we assume that $I_2 = \bar{I}_2$.

Equations (7) and the corresponding equations for a linear fluid [10] differ in terms containing the second viscosity μ_2.

Here, the role of pressure is played by the non-isotropic shear-dependent function p_*. When $i = j$ we obtain the balance equations for the fluctuation energy per unit volume in the i direction

$$\frac{\rho}{2}\left(\frac{\partial \overline{u_i^2}}{\partial t} + \overline{U}_a \frac{\partial \overline{u_i^2}}{\partial x_a}\right) + \rho\, \overline{u_i u_a} \frac{\partial}{\partial x_a}\left(\overline{U}_i - \frac{1}{\rho}\mu_2 \frac{\partial \overline{I}_2}{\partial x_i}\right) - \overline{p_* \frac{\partial u_i}{\partial x_i}} + (\mu - \mu_2 \overline{I}_2)\times \overline{\left(\frac{\partial u_i}{\partial x_a}\right)^2} + \frac{\partial}{\partial x_a}\left[\overline{u_a\left(\frac{1}{2}\rho u_i^2 + p_* \delta_{ia}\right)} - \frac{1}{2}(\mu - \mu_2 \overline{I}_2)\frac{\partial \overline{u_i^2}}{\partial x_a}\right] - \overline{u_i f_i} = 0. \quad (8)$$

The physical significance of the terms remains as before [8], but the dissipative term has a more complicated form.

As before [9, 10], we assume the approximate validity of the Kolmogorov hypothesis [7], i.e., the dissipation per unit mass of fluid and other characteristic quantities depend only on the turbulent energy $\overline{E}$ and the turbulence scale l.*

We make the following semiempirical approximations for the dissipative term and for the second moments "pressure-spatial velocity derivatives" [9]

$$(\nu - \nu_2 \overline{I}_2)\overline{\left(\frac{\partial u_i}{\partial x_a}\right)^2} = \frac{c}{3}\frac{\overline{E}^{\frac{3}{2}}}{l} + (\nu - \nu_2 \overline{I}_2)\frac{c_1}{2}\frac{\overline{u_i^2}}{l^2},$$

$$2(\nu - \nu_2 \overline{I}_2)\overline{\frac{\partial u_i}{\partial x_a}\frac{\partial u_j}{\partial x_a}} = \frac{2}{3}c\,\delta_{ij}\frac{\overline{E}^{\frac{3}{2}}}{l} + (\nu - \nu_2 \overline{I}_2)c_1 \frac{\overline{u_i u_j}}{l^2}, \quad (9)$$

$$\frac{1}{\rho}\overline{p_* \frac{\partial u_i}{\partial x_i}} = -k\frac{\overline{E}^{\frac{1}{2}}}{l}\left(\frac{\overline{u_i^2}}{2} - \frac{\overline{E}}{3}\right),$$

$$\frac{1}{\rho}\overline{p_*\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} = -\frac{\overline{E}^{\frac{1}{2}}}{l}k\left(\overline{u_i u_j} - \frac{2}{3}\delta_{ij}\overline{E}\right). \quad (10)$$

Constants c and c_1 are obtained from Laufer's experiments [17], while constant k must be determined experimentally for fluids with different values of μ_2. As Laufer's experiments show, turbulent diffusion of pulsation energy is important only near the axis, i.e., in a flow region of secondary importance for many problems owing to the fullness of the turbulent profile. Disregarding this flow region, we neglect the corresponding terms in Eqs. (7). After substituting relations (9) and (10) into Eqs. (7), we obtain a system of second-moment balance equations (without taking external forces into account)

$$\frac{\partial \overline{u_i u_j}}{\partial t} + \overline{U}_a \frac{\partial}{\partial x_a}\overline{u_i u_j} + \overline{u_j u_a}\frac{\partial}{\partial x_a}\left(\overline{U}_i - \nu_2 \frac{\partial \overline{I}_2}{\partial x_i}\right) + \overline{u_i u_a}\frac{\partial}{\partial x_a}\left(\overline{U}_j - \nu_2\frac{\partial \overline{I}_2}{\partial x_j}\right) + \frac{\overline{E}^{\frac{1}{2}}}{l}k\left(\overline{u_i u_j} - \frac{2}{3}\delta_{ij}\overline{E}\right) + \frac{2}{3}c\,\delta_{ij}\frac{\overline{E}^{\frac{3}{2}}}{l} + (\nu - \nu_2 \overline{I}_2)c_1\frac{\overline{u_i u_j}}{l^2} - \frac{\partial}{\partial x_a}\left[(\nu - \nu_2 \overline{I}_2)\frac{\partial \overline{u_i u_j}}{\partial x_a}\right] = 0, \quad (11)$$

together with the Reynolds equations, which in our case have the form

$$\rho\left(\frac{\partial \overline{U}_i}{\partial t} + \overline{U}_a \frac{\partial \overline{U}_i}{\partial x_a} + \overline{u_a \frac{\partial u_i}{\partial x_a}}\right) = -\frac{\partial \overline{p}}{\partial x_i} + (\mu - \mu_2 \overline{I}_2)\frac{\partial}{\partial x_a}\left(\frac{\partial \overline{U}_i}{\partial x_a} + \frac{\partial \overline{U}_a}{\partial x_i}\right) - \mu_2 \frac{\partial \overline{I}_2}{\partial x_a}\left(\frac{\partial \overline{U}_i}{\partial x_a} + \frac{\partial \overline{U}_a}{\partial x_i}\right),$$

$$\frac{\partial \overline{U}_a}{\partial x_a} = 0. \quad (12)$$

This system can be used to determine the principal mean and fluctuation characteristics of turbulent pipe flows. It should be kept in mind that system (11)–(12) is closed with respect to the first and second moments, but the problem of calculating the flow characteristics as functions of the coordinates can be completely solved only if the scale of turbulence l, which, generally speaking, is a function of the principal invariants of the strain rate tensor, is specified.

A special feature of this method (using the equations for the change of Reynolds stresses to close the system of Reynolds equations) as compared with the method based on the Prandtl-Boussinesq hypothesis is that much more information on the nature of the fluctuation component can be obtained since the fluctuation characteristics of the flow are calculated directly. Moreover, it is possible to make fuller allowance for the effect of the fluctuating motion on the mean flow characteristics. Thus, for example, in the case of quasi-plane turbulent pipe flow, by means of the Prandtl-Boussinesq hypothesis we can approximate only the shear stresses $-\overline{uv} = l^2|d\overline{U}/dy|d\overline{U}/dy$, without either obtaining information on the normal stresses or a quantitative estimate of their direct effect on the integral characteristics. We can also calculate completely the single-point second moments $\overline{uv}$, $\overline{uw}$, $\overline{vw}$, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$, and the fluctuation energy $\overline{E}$ and show their effect on the mean-flow characteristics. From Eqs. (11) and (12), neglecting the effect of viscous diffusion on energy transfer and using the dimensionless

*The scale problem is not considered in this paper.

quantities R_l,* R_E, R_*, and N, we obtain the following equations for the principal fluctuation characteristics:

$$\overline{uw} = \overline{vw} = 0; \tag{13}$$

$$\frac{\sqrt{\overline{v^2}}}{v_*} = \frac{R_E}{\frac{l}{a}R_*}\left(\frac{2}{3}\right)^{1/2}\times \left[\frac{(k-c)R_E}{2l^2 N \frac{d^2}{dy^2}\overline{R}_l^2 + kR_E + c_1(1+N\overline{R}_l^2)}\right]^{\frac{1}{2}}; \tag{14}$$

$$\frac{\sqrt{\overline{w^2}}}{v_*} = \left[\frac{2}{3}\,\frac{(k-c)R_E}{kR_E + c_1(1+N\overline{R}_l^2)}\right]^{\frac{1}{2}}\frac{R_E}{\frac{l}{a}R_*}; \tag{15}$$

$$-\frac{\overline{uv}}{v_*^2} = \frac{R_E^2}{R_l}\left[c_1(1+N\overline{R}_l^2) + cR_E + \frac{2}{3}l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 \times \frac{(k-c)R_E}{2l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 + kR_E + c_1(1+N\overline{R}_l^2)}\right]\frac{1}{\left(\frac{l}{a}\right)^2 R_*}; \tag{16}$$

$$\frac{\sqrt{\overline{u^2}}}{v_*} = \left\{2\left[1 - \frac{2}{3}(k-c)R_E \times \frac{l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 + kR_E + c_1(1+N\overline{R}_l^2)}{\left[2l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 + kR_E + c_1(1+N\overline{R}_l^2)\right]}\times \frac{1}{[kR_E + c_1(1+N\overline{R}_l^2)]}\right]\right\}^{\frac{1}{2}}\frac{R_E}{\frac{l}{a}R_*}; \tag{17}$$

$$\frac{\sqrt{\overline{E}}}{v_*} = \frac{R_E}{\frac{l}{a}R_*}. \tag{18}$$

From Eq. (16) the following relation is obtained for the turbulent and molecular transfer coefficients

$$\frac{\varepsilon}{\nu} = \frac{-\overline{uv}}{\nu\frac{d\overline{U}}{dy}} = \frac{R_E^2}{R_l^2}\left[c_1(1+N\overline{R}_l^2) + cR_E + \frac{2}{3}l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 \times \frac{(k-c)R_E}{2l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 + kR_E + c_1(1+N\overline{R}_l^2)}\right]. \tag{19}$$

From Eqs. (16), (17), and (14) the single-point correlation coefficient for the horizontal and vertical velocity fluctuations is found to be

$$R_{\overline{uv}} = \frac{\overline{uv}}{\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}}. \tag{20}$$

In our case, correct to R_E^2 the fluctuation energy balance equation can be expressed in the following dimensionless form:

$$\frac{2}{3}\,\frac{(k-c)R_l^2 R_E}{\left[2l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 + kR_E + c_1(1+N\overline{R}_l^2)\right]}\times \frac{1}{\left[l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 + kR_E + c_1(1+N\overline{R}_l^2)\right]} =$$

$$= c_1(1+N\overline{R}_l^2) + cR_E + l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 \times \frac{\frac{2}{3}(k-c)R_E}{2l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 + kR_E + c_1(1+N\overline{R}_l^2)}. \tag{21}$$

The first integral of the mean-motion equation (12), written in terms of dimensionless complexes, reduces to

$$R_l(1+N\overline{R}_l^2) + \frac{R_E^2}{R_l}\left[c_1(1+N\overline{R}_l^2) + cR_E + l^2 N^2\frac{d^2}{dy^2}\overline{R}_l^2 \times \frac{\frac{2}{3}(k-c)R_E}{2l^2 N\frac{d^2}{dy^2}\overline{R}_l^2 + kR_E + c_1(1+N\overline{R}_l^2)}\right] =$$

$$= \left(\frac{l}{a}\right)^2 R_*^2\left(1 - \frac{y}{a}\right). \tag{22}$$

With the numerical solution of Eq. (21) for given empirical constants $R_E = R_E(R_l, N)$ can be determined. Using this and specifying l, from (22) we can find the local Reynolds number $R_l = R_l(l, N)$ for given R_*. Knowing R_l and R_E, from (14)–(20) we find the unknown fluctuation characteristics and also, with a view to determining the local Reynolds number, we find the mean velocity distribution by integrating the equation

$$\frac{d\overline{U}}{dy} = \frac{\nu}{l^2}R_l. \tag{23}$$

Considering only the turbulent core of the flow, where R_E and R_l are large, at small Reynolds numbers we neglect the viscous dissipation of fluctuation energy as compared with the Kolmogorov dissipation, i.e., terms containing c_1. Here, we assume that

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*$R_l = (l^2/\nu)(d\overline{U}/dy)$ is the local Reynolds number first introduced by L. G. Loitsyanskii [18], $\overline{R}_l = R_l/l^2$.

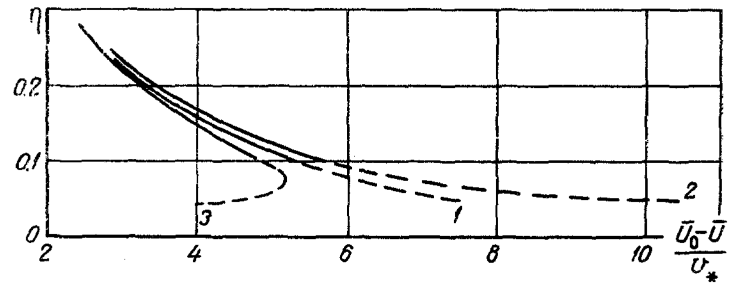

Fig. 1. Velocity defect curves for: 1) $M/R_*^3 = 0$; 2) 10^{-5}; 3) -10^{-5}.

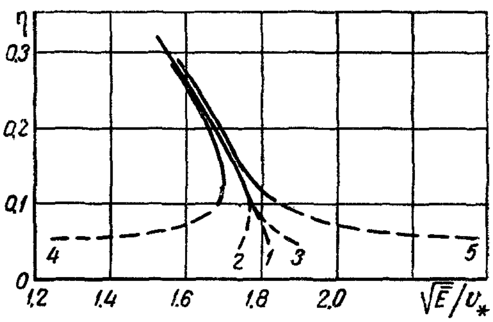

Fig. 2. Variation of the energy of the fluctuating components for: 1) $M/R_*^3 = 0$; 2) 10^{-5}; 3) -10^{-5}; 4) 10^{-4}; 5) -10^{-4}.

the dissipation associated with shear viscosity at least does not exceed the dissipation associated with ordinary viscosity.

We have to evaluate the coefficient k. To simplify the calculations, we take the Prandtl mixing length as the scale l, using it only to estimate k. Then the expression for the turbulent friction with the assumed constraints

$$-\rho\,\overline{uv} = \rho l^2 \left|\frac{d\bar U}{dy}\right| \frac{d\bar U}{dy} \frac{c}{k^3} \left[\frac{1 + \frac{2}{k}\frac{l^2}{\mathrm{R}_E} \mathrm{N} \frac{d^2}{dy^2} \bar{\mathrm{R}}_l^2}{\frac{2}{3}\left(\frac{k}{c}-1\right)} \times \left(1 + \frac{1}{k}\frac{l^2}{\mathrm{R}_E}\mathrm{N}\frac{d^2}{dy^2}\bar{\mathrm{R}}_l^2\right) \times \left(1 + \frac{l^2}{\mathrm{R}_E}\mathrm{N}\frac{d^2}{dy^2}\bar{\mathrm{R}}_l^2 \frac{\frac{2}{3}(k-c)}{1 + \frac{2}{k}\frac{l^2}{\mathrm{R}_E}\mathrm{N}\frac{d^2}{dy^2}\bar{\mathrm{R}}_l^2}\right)^{\frac{1}{3}}\right]^{-\frac{3}{2}}$$

should coincide at N = 0 with the Prandtl formula, whence follows the condition that must be satisfied by the coefficients k and c at $l = \varkappa y$:

$$\left[\frac{2}{3}\left(\frac{k}{c}-1\right)\right]^{\frac{3}{2}} \frac{c}{k^3} = 1. \qquad (24)$$

We will consider the motion of a fluid for which $\mathrm{N}/\mathrm{R}_E \ll 1$. In this case, to calculate the unknown characteristics of the mean and fluctuating motion we employ the method of successive approximations.

In the zero-order approximation (with N = 0) the characteristics coincide with those obtained by Levin [10] for a linear plane-channel flow. Then, confining ourselves to the first powers of the parameter N, in first approximation the unknown characteristics are given by

$$\frac{\sqrt{\overline{u^2}}}{v_*} = \left(\frac{\sqrt{\overline{u^2}}}{v_*}\right)_0 \left\{1 + F(M)\frac{1}{6\left(2+\frac{k}{c}\right)} \times \left[(3-\eta)\left(1+2\frac{k}{c}\right)\left(10-\frac{k}{c}\right)\right]\right\},$$

$$\frac{\sqrt{\overline{v^2}}}{v_*} = \left(\frac{\sqrt{\overline{v^2}}}{v_*}\right)_0 \times \left\{1 + F(M)\frac{1}{6}\left[-(3-\eta)\left(1+2\frac{k}{c}\right)\right]\right\},$$

$$\frac{\sqrt{\overline{w^2}}}{v_*} = \left(\frac{\sqrt{\overline{w^2}}}{v_*}\right)_0 \times \left\{1 + F(M)\frac{1}{6}\left[(3-\eta)\left(11-2\frac{k}{c}\right)\right]\right\},$$

$$\frac{\varepsilon}{\nu} = \left(\frac{\varepsilon}{\nu}\right)_0 \left\{1 - F(M)\frac{1}{6}\left[(3-\eta)\left(25+2\frac{k}{c}\right)\right]\right\},$$

$$R_{\overline{uv}} = (R_{\overline{uv}})_0 \left\{1 - F(M)\frac{1}{3\left(2+\frac{k}{c}\right)} \times \left[(3-\eta)\left(1+2\frac{k}{c}\right)\left(4-\frac{k}{c}\right)\right]\right\},$$

$$\frac{\sqrt{\bar E}}{v_*} = \left(\frac{\sqrt{\bar E}}{v_*}\right)_0 \times \left\{1 + F(M)\frac{1}{6}\left[(3-\eta)\left(11-2\frac{k}{c}\right)\right]\right\},$$

$$\frac{d\left(\frac{\bar U}{v_*}\right)}{d\eta} = \left(\frac{d\left(\frac{\bar U}{v_*}\right)}{d\eta}\right)_0 \times \left\{1 + F(M)\frac{1}{6}\left[(3-\eta)\left(25+2\frac{k}{c}\right)\right]\right\}, \qquad (25)$$

where $F(M) = (M/R_*^3)(c^{1/3}/k\varkappa)[1/\eta^3(1-\eta)^{1/2}]$, $M = (\mu_2/\mu^3)\tau_W$, $\eta = yv_*/\nu$ is a universal coordinate.

In first approximation, Eqs. (21) and (22) have the form

$$\mathrm{R}_E = \mathrm{R}_{E_0}\left\{1 + F(M)\frac{1}{6}\left[(3-\eta)\left(11-2\frac{k}{c}\right)\right]\right\},$$

$$\mathrm{R}_l = \mathrm{R}_{l_0}\left\{1 + F(M)\frac{1}{6}\left[(3-\eta)\left(25+2\frac{k}{c}\right)\right]\right\}. \qquad (26)$$

The calculation process can be extended to any number of successive approximations. We limit ourselves to the first approximation. Calculated curves showing the kinetic energy of the fluctuating components and the velocity defect in the flow core are presented in Figs. 1 and 2. From these figures it is clear that the over-all viscosity effect reduces either to the additional dissipation ($\nu_2 > 0$) or to the additional generation ($\nu_2 < 0$) of pulsation energy. Accordingly, the mean velocity profile is either laminarized or becomes fuller. These effects are manifested strongly in the lower part of the turbulent core and extremely weakly in the central region (at least at the calculated values of the parameter M/R_*^3).

For very small η the theoretical results presented should be treated cautiously, since at the values of M/R_*^3 in question the first approximation for calculating the characteristics is clearly inadequate near the walls and, accordingly, the parts of the curves near the walls are merely illustrative.

NOTATION

τ_{ij} and $\dot e_{ij}$ are the stress tensor and the strain-rate tensor; I is the symbol common to the three invariants of $\dot e_{ij}$; δ_{ij} is the Kronecker delta; $\bar U$ is the mean velocity; u, v, and w are the velocity fluctuation components; x_i are the Cartesian coordinates; ρ, t, and F_i are the density, time, and body force components, respectively; k, c, and c_1 are the empirical constants determined from Laufer's experiments [17]; $N = \nu\nu_2/a^4$, where a is the half-width of the channel; τ_w is the wall friction; $R_E = l(\overline{E})^{1/2}/\nu$ is the energy Reynolds number; $R_* = v_* a/\nu$ is the dynamic Reynolds number.

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